QUESTION BANK(for DA students)

MATHS-10+2

Relations & Functions

Multiple Choice Questions:-

1. Let $\boldsymbol{R}$ be the relation in the set $\boldsymbol{N}$ of Natural number given by $\boldsymbol{R} = \{(x, y): x = y - 2,\ y > 6\}$ Choose the correct answer.
(a) $(2,4) \in \boldsymbol{R}$ (b) $(3,8) \in \boldsymbol{R}$ (c) $(6,8) \in \boldsymbol{R}$ (d) $(8,7) \in \boldsymbol{R}$
Answer: (c) $(6,8) \in \boldsymbol{R}$

2. Let R = {(1,2), (2,2) ,(1,1),(4,4), (1,3),(3,3),(3,2)} be a relation defined on the set A= {1,2,3,4}, then

(a) $\boldsymbol{R}$ is reflexive and symmetric but not transitive
(b) $\boldsymbol{R}$ is reflexive and transitive but not symmetric
(c) $\boldsymbol{R}$ is symmetric and transitive but not reflexive
(d) $\boldsymbol{R}$ is an equivalence relation
Answer: (b) $\boldsymbol{R}$ is reflexive and transitive but not symmetric

3. Let $f: \boldsymbol{N} \to \boldsymbol{N}, f(x) = x^2$, then

(a) f is only one-one but not onto
(b) f is only onto but not one-one
(c) f is one-one and onto
(d) None of the above
Answer: (a) f is only one-one but not onto

4. $If\ \ f: \boldsymbol{R} - \{0\} \to \boldsymbol{R} - \{0\},\ f\ (x) = \frac{1}{x}\ then\ fof\ (x)\ \ is$

(a) 1 (b) $\frac{1}{x}$ (c) x (d) none of these
Answer: (c) x

5. If $\boldsymbol{R} = \{(x, y): x - y\ is\ divisible\ by\ 3, x, y \in \mathrm{Z}\}$ then $\boldsymbol{R}$ is

(a) Reflexive only (b) Symmetric only
(c) Transitive only (d) Equivalence Relation
Answer: (d) Equivalence Relation

6. If $f(x) = \sin x, g(x) = x^2\ then\ \ fog(x)$ is equal to

(a) $\sin(x^2)$ (b) $(\sin x)^2$ (c) $(\sin x)^x$ (d) x
Answer: (a) $\sin(x^2)$

Match the column

7.

	Column-A	Column-B
(i)	$f: A \xrightarrow{one-one} f\ (A)$	(a) f is one one only
(ii)	$f: \boldsymbol{N} \to \boldsymbol{N}, f\ (x) = x^2$	(b) f is onto only
		(c) $f^{-1}\ exists$

(**Answer:-** $\boldsymbol{i.} \to (\boldsymbol{c})$, $\boldsymbol{ii.} \to (\boldsymbol{a})$)

8.

	Column-A	Column-B
(i)	$f\ (x) = \ x\ \forall\ \boldsymbol{R}$	(a) f, g are inverse of each other
(ii)	$fog(x) = \ x\ \forall\ \boldsymbol{R}$	(b) Identity function
		(c) $f = \ g$
		(d) constant function

(**Answer:-** $\boldsymbol{i.} \rightarrow (\boldsymbol{b})$, $\boldsymbol{ii.} \rightarrow (\boldsymbol{a})$)

9.

	Column-A	Column-B
(i)	$\boldsymbol{R} = \{(x, y): x \leq y^2,\ x, y \epsilon\ \boldsymbol{R}\}$	(a) Equivalence Relation
(ii)	$\boldsymbol{R} = \{(x, y): x - y\ is\ an\ integer\ x, y \in \mathrm{Z}\}$	(b) Only symmetric
		(c) Not reflexive

(**Answer:-** $\boldsymbol{i.} \rightarrow (\boldsymbol{c})$, $\boldsymbol{ii.} \rightarrow (\boldsymbol{a})$)

10.

	Column-A	Column-B
(i)	$f\ (x) = logx, g(x) = e^x$	(a) $fog\ (x) = \sin(logx)$
(ii)	$f\ (x) = sinx, g(x) = logx$	(b) $gof\ (x) = \sin(logx)$
		(c) $fog\ (x) = x$

(**Answer:-** $\boldsymbol{i.} \rightarrow (\boldsymbol{c})$, $\boldsymbol{ii.} \rightarrow (\boldsymbol{a})$)

11.

	Column-A	Column-B
(i)	$R = \Phi$	(a) Universal Relation
(ii)	$R = A\ \times A$	(b) R is reflexive but not symmetric
		(c) Empty Relation

(**Answer:-** $\boldsymbol{i.} \rightarrow (\boldsymbol{c})$, $\boldsymbol{ii.} \rightarrow (\boldsymbol{a})$)

12.

	Column-A	Column-B
(i)	$R = \{(\mathrm{x}, \mathrm{x}): \mathrm{x} \in \mathrm{A}\}$	(a) Empty Relation
(ii)	$R = \{(1,2), (2,3), (1,3)\}$	(b) Identity Relation
		(c) Transitivity

(**Answer:-** $\boldsymbol{i.} \rightarrow (\boldsymbol{b})$, $\boldsymbol{ii.} \rightarrow (\boldsymbol{c})$

Fill in the blanks from the following options:-

(one-one, onto, 2,3, $A\ \times A$, reflexive, symmetric, 1, $\sqrt{3}$, $\frac{1}{\sqrt{3}}$)

13. If $f\ (x) = logx\ \ then\ f(e)$..... **Answer:** 1
14. If $f\ (x) = tanx\ \ then\ f\ \left(\frac{\pi}{3}\right)$..... **Answer:** $\sqrt{3}$
15. Identity relation is also ..... **Answer:** reflexive
16. If R is defined relation on set A then R is subset of..... **Answer:** $A\ \times A$
17. If $f\ (x) = \ |x|$ and $g(x) = [x]$ then fog (2.5)= ..... **Answer:** 2
18. If f is defined as $f : A\ \rightarrow f(A)$ then f is ..... **Answer:** onto

State as true or false:

19. Inverse of a function exists if and only if function in one-one and onto. (✓)
20. If $f : A\ \rightarrow B$ and $g : C \rightarrow D$ then fog is defined. (×)
21. If $fof\ (x) = x$ then $f = f^{-1}$ (✓)

22. If $f\ (x) = \ x^3$ and $\ g\ (x) = \ x^{\frac{1}{3}}$ then fog $(2) = \ 0$ (×)

23. If R is any relation defined on A where A is non-empty set then $R \subseteq A \times A$ (✓)

24. If $R = \{\ (x\ , y)$: $x - y\ is\ an\ integer, x, y\ \in \boldsymbol{Z}\ \}$ is defined on set of integers then R is not reflexive. (×)

25. If $f(x) = \ [x]\ \ then\ f\ (2.5) = \ -2$ (×)

26. If $f(x) = \ |x|\ \ then\ f\ (-7.5) = \ 7.5$ (✓)

INVERSE TRIGONOMETRIC FUNCTION

Multiple Choice Questions:-

1. Principal value of $\sin^{-1}\left(\frac{-1}{2}\right)$ is :

   (a) $-\frac{\pi}{6}$ (b) $\frac{\pi}{6}$ (c) $-\frac{\pi}{3}$ (d) $\frac{\pi}{3}$

   Answer: (a) $-\frac{\pi}{6}$

2. Principal value of $\cos^{-1}\left(\frac{-1}{2}\right)$ is :

   (b) $\frac{\pi}{3}$ (b) $\frac{2\,\pi}{3}$ (c) $-\frac{\pi}{3}$ (d) $\frac{5\,\pi}{3}$

   Answer: (b) $\frac{2\,\pi}{3}$

3. Principal value of $\tan^{-1}\left(\sqrt{3}\right)$ is :

   (a) 0 (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

   Answer: (d) $\frac{\pi}{3}$

4. If $\sin\ (\sin^{-1}\frac{1}{5} + \ \cos^{-1} x\ ) = 1$ then value of x is:

   (a) 5 (b) $\frac{1}{5}$ (c) -5 (d) $\frac{-\,1}{5}$

   Answer: (b) $\frac{1}{5}$

5. If $\sin^{-1} x = y$, then:

   (a) $x \in [-1,1]$ (b) $x \in (-1,1)$ (c) $x \in [0,1]$ (d) $x \in (0,1)$

   Answer: $x \in [-1,1]$

6. If $\tan^{-1} x = y$, then:

   (a) $y \in \boldsymbol{R}$ (b) $y \in [\frac{-\pi}{2}, \frac{\pi}{2}]$ (c) $y \in (\frac{-\pi}{2}, \frac{\pi}{2})$ (d) $y \in [-1,1]$

   Answer: (c) $y \in (\frac{-\pi}{2}, \frac{\pi}{2})$

7. $\tan^{-1}\left(\sqrt{3}\right) - \sec^{-1}(-2)$ is equal to

   (a) π (b) $\frac{-\pi}{3}$ (c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$

   Answer: (b) $\frac{-\pi}{3}$

Match the Column:

8.

Column-A	Column-B
(i) $cos^{-1}\left[\cos\left(\frac{\pi}{6}\right)\right]$	(a) $3\ cos^{-1}x$
(ii) $sin^{-1}(3x - 4x^3)$	(b) $3\ sin^{-1}x$
	(c) $\pi\ /6$

(**Answer:-** $\boldsymbol{i.} \rightarrow (\boldsymbol{c})$, $\boldsymbol{ii.} \rightarrow (\boldsymbol{b})$)

9. Column-A Column-B

(i)	$sin^{-1}x$	(a) $Domain = [\,-1, 1]$
(ii)	$cos^{-1}x$	(b) $Range = [\,0,\ \frac{\pi}{2}\,]$
		(c) $Range = [\,\frac{-\pi}{2}, \frac{\pi}{2}\,]$

(**Answer:-** $\boldsymbol{i.} \rightarrow (\boldsymbol{c})$, $\boldsymbol{ii.} \rightarrow (\boldsymbol{a})$)

10.

	Column-A	Column-B
(i)	$tan^{-1}\,(1)$	(a) $-\frac{\pi}{3}$
(ii)	$sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$	(b) $\frac{\pi}{3}$
		(c) $\frac{\pi}{4}$

(**Answer:-** $\boldsymbol{i.} \rightarrow (\boldsymbol{c})$, $\boldsymbol{ii.} \rightarrow (\boldsymbol{a})$)

11.

	Column-A	Column-B
(i)	$sin^{-1}\left(\frac{2x}{1+x^2}\right)$	(a) $cos^{-1}x$
(ii)	$sin^{-1}(-x)$	(b) $2\ tan^{-1}x$
		(c) $-sin^{-1}x$

(**Answer:-** $\boldsymbol{i.} \rightarrow (\boldsymbol{b})$, $\boldsymbol{ii.} \rightarrow (\boldsymbol{c})$)

12.

	Column-A	Column-B
(i)	$tan^{-1}x + tan^{-1}y$	(a) $tan^{-1}\left(\frac{x+y}{1-xy}\right)$
(ii)	$tan^{-1}x - tan^{-1}y$	(b) $tan^{-1}\left(\frac{x-y}{1+xy}\right)$
		(c) $tan^{-1}\left(\frac{x+y}{1+xy}\right)$

(**Answer:-** $\boldsymbol{i.} \rightarrow (\boldsymbol{a})$, $\boldsymbol{ii.} \rightarrow (\boldsymbol{b})$)

13.

	Column-A	Column-B
(i)	$cos^{-1}\,(4x^3 - 3x)$	(a) $2\ tan^{-1}x$
(ii)	$tan^{-1}\left(\frac{2x}{1-x^2}\right)$	(b) $3\ tan^{-1}x$
		(c) $3\ cos^{-1}x$

(**Answer:-** $\boldsymbol{i.} \rightarrow (\boldsymbol{c})$, $\boldsymbol{ii.} \rightarrow (\boldsymbol{a})$)

Fill in the blanks from the following options:-

$$\left(\left(\frac{-\pi}{2}, \frac{\pi}{2}\right),\ \frac{\pi}{2}\ ,\ [0,\ \pi\,],\ \left[\frac{-\pi}{2}, \frac{\pi}{2}\right],\ \sqrt{3}\ ,\ \boldsymbol{R},\ \frac{-1}{2},\ \bar{\sqrt{3}}\ \right)$$

14. $sin^{-1}x + cos^{-1}x =$ …… **Answer:** $\frac{\pi}{2}$
15. If $cos^{-1}x = y\ then\ y \in$ …… **Answer:** $[0,\ \pi\,]$
16. If $tan^{-1}x = y\ then\ x \in$ …… **Answer:** $\boldsymbol{R}$
17. If $tan^{-1}x = \frac{\pi}{3}\ then\ x =$ …… **Answer:** $\sqrt{3}$
18. If $cos^{-1}x = \frac{2\,\pi}{3}\ then\ x =$ …… **Answer:** $\frac{-1}{2}$
19. If $sin^{-1}(\sin x)\ = x\ then\ x \in$ …… **Answer:** $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

State as true or false:

20. Principal value of $sin^{-1}\left(\frac{1}{2}\right)\ is\ \frac{\pi}{3}$ (×)

21. Principal value of $tan^{-1}(-1)\ is\ \frac{\pi}{4}$ (×)

22. If $\theta = sin^{-1}\left(\frac{3}{5}\right)$ then $\theta = tan^{-1}\left(\frac{4}{5}\right)$ (×)

23. If $sin^{-1}x = y$ then $y \in [0,\ \pi]-\left\{\frac{\pi}{2}\right\}$ (×)

24. Simplest form of $tan^{-1}\left(\frac{1-tanx}{1+tanx}\right)\ is\ \frac{\pi}{4} - x$ (✓)

25. Simplest form of $sin^{-1}(3x - 4x^3)\ is\ 3\ cos^{-1}x$ (×)

3 Marks Questions

Prove that:

1. $tan^{-1}\ \frac{1}{3} + tan^{-1}\ \frac{1}{5} = tan^{-1}\ \frac{4}{7}$

2. $tan^{-1}\ \frac{1}{2} + tan^{-1}\ \frac{2}{11} = tan^{-1}\ \frac{3}{4}$

3. $2tan^{-1}\ \frac{1}{2} + tan^{-1}\ \frac{1}{7} = tan^{-1}\ \frac{31}{17}$

4. $sin^{-1}\left(\frac{3}{5}\right) - sin^{-1}\ \left(\frac{8}{17}\right) = cos^{-1}\ \left(\frac{84}{85}\right)$

5. $2sin^{-1}\ \frac{3}{5} = tan^{-1}\ \frac{24}{7}$

6. $tan^{-1}\ \sqrt{x} = \frac{1}{2}cos^{-1}\ \left(\frac{1-x}{1+x}\right)$

7. Solve: $tan^{-1}\ 2x + tan^{-1}\ 3x = \frac{\pi}{4}$ **Answer:** $\left(-1 \text{ or } \frac{1}{6}\right)$

8. Solve: $2\ tan^{-1}\ (\cos x) = tan^{-1}\ (2\ cosec\ x)$ **Answer:** $\left(\frac{\pi}{4}\right)$

9. If $tan^{-1}\ \frac{x-1}{x-2} + tan^{-1}\ \frac{x+1}{x+2} = \frac{\pi}{4}$, then find the value of x

Answer: $\left(\frac{\pm 1}{\sqrt{2}}\right)$

10. If $tan^{-1}\ \frac{x}{2} + tan^{-1}\ \frac{x}{3} = \frac{\pi}{4}$, then find the value of x

Answer: $(1 \text{ or } -6)$

11. Simplify: $tan^{-1}\left[\frac{\mathrm{a}\cos x - b\sin x}{b\cos x + a\sin x}\right]$

Answer: $\left\{\tan^{-1}\left(\frac{\mathrm{a}}{\mathrm{b}}\right) - \mathrm{x}\right\}$

12. Simplify: $\tan\frac{1}{2}\left[sin^{-1}\ \frac{2x}{1+x^2} + \ cos^{-1}\frac{1-y^2}{1+y^2}\right]$

Answer: $\left(\frac{x+y}{1-xy}\right)$

MATRICES

Multiple Choice Questions:-

1. If matrix $A = \left[a_{ij}\right]_{2\times 2}$ is such that $a_{ij} = \ i^2$ + j then a_{21} is

(a) 4 (b) 5 (c) 6 (d) 7

Answer:(b) 5

2. If $AB = C$ where A is matrix of order 2×3 and B is a matrix of order 3×4 then order of matrix C is:

(a) 2×4 (b) 4×2 (c) 2×2 (d) 3×3

Answer:(a) 2×4

3. If $A + B = C$ where order of matrices A and B is 3×4 then order of matrix c is

(a) 4×3 (b) 3×2 (c) 2×3 (d) 3×4

Answer:(d) 3×4

4. If matrix $AB = C$ where B is a matrix of order 4×2 and C is a matrix of order 3×2 then order of matrix A is:

(a) 3×4 (b) 4×3 (c) 3×3 (d) 2×2

Answer: (a) 3×4

5. The number of all possible matrices of order 3×3 with entry 0 or 1 is:(a) 27

(b) 18 (c) 81 (d) 512

Answer: (c) 81

6. If $\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$ then

(a) $x = \frac{-1}{3}, y = 7$ (b) $Not\ possible\ to\ find\ the\ value\ of\ x\ \&\ y$

(c) $x = \frac{-2}{3}, y = 7$ (d) $x = \frac{-1}{3},\ y = \frac{-2}{3}$

Answer: (b) $Not\ possible\ to\ find\ the\ value\ of\ x\ \&\ y$

Match the column

7.

	Column-A	Column-B
(i)	$A + A'$	(a) Rectangular Matrix
(ii)	$A - A'$	(b) skew-symmetric matrix
		(c) Symmetric matrix

(**Answer:-** $\boldsymbol{i.} \rightarrow (\boldsymbol{c})$, $\boldsymbol{ii.} \rightarrow (\boldsymbol{b})$)

8.

	Column-A	Column-B
(i)	$(AB)'$	(a) $A'B'$
(ii)	$(BA)'$	(b) $(A+B)'$
		(c) $B'A'$

(**Answer:-** $\boldsymbol{i.} \rightarrow (\boldsymbol{c})$, $\boldsymbol{ii.} \rightarrow (\boldsymbol{a})$)

9.

	Column-A	Column-B
(i)	$(AB)^{-1}$	(a) A
(ii)	$(A')'$	(b) A'
		(c) $B^{-1}A^{-1}$
		(d) $A^{-1}B^{-1}$

(**Answer:-** $\boldsymbol{i.} \rightarrow (\boldsymbol{c})$, $\boldsymbol{ii.} \rightarrow (\boldsymbol{a})$)

10

	Column-A	Column-B
(i)	Identity Matrix	(a) Only one col.mn
(ii)	Row Matrix	(b) Square Matrix
		(c) Only one row

(**Answer:-** $\boldsymbol{i.} \rightarrow (\boldsymbol{b})$, $\boldsymbol{ii.} \rightarrow (\boldsymbol{c})$)

11.

	Column-A	Column-B

(i)	Matrix Addition	(a)	Non- commutative
(ii)	Matrix Multiplication	(b)	Transpose
		(c)	Commutative

(**Answer:-** $\boldsymbol{i.} \rightarrow (\boldsymbol{c})$, $\boldsymbol{ii.} \rightarrow (\boldsymbol{a})$)

12.

	Column-A		Column-B
(i)	$A = [a_{ij}]_{m\times n}, m = n$	(a)	Row Matrix
(ii)	$A = [a_{ij}]_{1\times n}$	(b)	Column Matrix
		(c)	Square Matrix

(**Answer:-** $\boldsymbol{i.} \rightarrow (\boldsymbol{c})$, $\boldsymbol{ii.} \rightarrow (\boldsymbol{a})$)

Fill in the blanks from the following options:-

(Inverse, 9,10, 3, symmetric, skew-symmetric, 4×3 , 4×4)

1. If $A = [a_{ij}]_{3\times4}$ such that $a_{ij} = i + 2j$ then a_{33} …….. **Answer:** 9
2. If order of matrix A is 5×2 then number of elements in A are …….. **Answer:** 10
3. If $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$, then z = …….. **Answer:** 3
4. If order of A is 3×4 then order of A' …….. **Answer:** 4×3
5. If for a matrix $A, A' = A$ holds then *A* is called …….. matrix. **Answer:** symmetric
6. If for a matrix $A, A' = -A$ holds then is called …….. matrix. **Answer:** skew-symmetric
7. If $AB = BA = I$ then *A* and *B* are …….. matrices of each other. **Answer:** Inverse

State as true or false:

8. If *A* and *B* are symmetric matrices of same order then $AB - BA$ is a symmetric matrix. (×)
9. If a matrix is symmetric as well as skew-symmetric then it is a null matrix. (✓)
10. Any square matrix can be expressed as the sum of symmetric and skew-symmetric matrix. (✓)
11. Matrix multiplication is not associative. (×)
12. AB is a null matrix iff either A is null matrix or B is null matrix. (×)
13. If *A* is a square matrix then $A - A'$ is skew-symmetric. (✓)

3 Marks Questions

1. If $A = [a_{ij}]_{2\times2}, a_{ij} = (i + 2j)^2$ then find A. **Answer:** $A = \begin{bmatrix} 9 & 25 \\ 16 & 36 \end{bmatrix}$
2. Find $x, y \text{ and } z, if \begin{bmatrix} x + y + z \\ x + z \\ y + z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

Answer: $(x = 2, y = 4,\ z = 3)$

3. Find X, if $Y = \begin{bmatrix} 5 & 3 \\ 2 & 6 \end{bmatrix} \text{ and } 2X + Y = \begin{bmatrix} 1 & -2 \\ 3 & 0 \end{bmatrix}$ **Answer:** $\boldsymbol{X} = \begin{bmatrix} -2 & -5/2 \\ 1/2 & -3 \end{bmatrix}$

4. If $A = \begin{bmatrix} 2 & 3 \\ 7 & 2 \end{bmatrix}$ then find $A^2 - 5A + 2\mathrm{I}$ **Answer:** $\boldsymbol{X} = \begin{bmatrix} 17 & -3 \\ -7 & 17 \end{bmatrix}$

5. If $A = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}, B = [-2 \;\; 3 \;\; 1]$ then verify that $(AB)' = B'A'$

6. Express $\begin{bmatrix} 5 & 6 \\ -1 & 7 \end{bmatrix}$ as sum of symmetric and skew-symmetric matrices.

7. If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ then show that $A + A'$ is a symmetric matrix.

8. If $A = \begin{bmatrix} 3 & -1 \\ 5 & 10 \end{bmatrix}$ then show that $A - A'$ is a skew-symmetric matrix.

9. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 \\ 2 & 5 \end{bmatrix}$ then show that $(A - B)' = A' - B'$

10. Find the inverse of $\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$ by elementary transformations.

Answer: $\begin{bmatrix} 1 & 0 \\ -1 & 1/_2 \end{bmatrix}$

11. If $A = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$ then find AB and BA.

Answer: $\mathrm{AB} = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix}, \mathrm{BA} = \begin{bmatrix} -10 & 2 & 21 \\ -16 & 2 & 37 \\ -2 - & 2 & 11 \end{bmatrix}$

12. Simplify : $\cos\theta \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$

Answer: $\mathrm{AB} = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$

4 Marks Questions

1. Give two examples each of

   (i) Row Matrix (ii) Square Matrix

2. Give two examples each of

   (i) Column Matrix (ii) Diagonal Matrix

3. Write null matrix and identity matrix of two different orders each.

4. If $\begin{bmatrix} x & y \\ z & a \\ b & c \end{bmatrix} = \begin{bmatrix} \frac{-3}{2} & 0 \\ 2 & \sqrt{6} \\ 3 & 2 \end{bmatrix}$ then find *x, y, z, a, b & c.*

5. Write two differences between symmetric matrices and skew-symmetric matrices.

6. Give two example each of

   (i) square matrix (ii) diagonal matrix

7. Give an example of matrices A and B where $A \neq 0, B \neq 0$ but $AB = 0$

8. Give an example of matrices A and B where $AB = BA$

9. Write two differences between null matrix and identity matrix.

10. Write two differences between identity matrix and diagonal matrix

DETERMINANTS

Multiple Choice Questions:

1. If $\begin{vmatrix} x & 1 \\ 2 & 1 \end{vmatrix} = \begin{vmatrix} 3 & 1 \\ 2 & 1 \end{vmatrix}$ then value of x is:
(a) 1 (b) 2 (c) 3 (d) 4

Answer: (c) 3

2. If $\begin{vmatrix} x & 1 \\ 1 & x \end{vmatrix} = \begin{vmatrix} 2 & 0 \\ 2 & 4 \end{vmatrix}$ then value of x is:
(a) 0 (b) ± 1 (c) ± 2 (d) ± 3
Answer: (d) ± 3

3. If $\Delta = \begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix}$ then value of Δ is:
(a) 18 (b) 20 (c) 22 (d) 24
Answer: (a) 18

4. Which of the following is correct:
(a) Determinant is a square matrix.
(b) Determinant is a number associated to a matrix.
(c) Determinant is a number associated to a square matrix.
(d) None of these
Answer: (c) Determinant is a number associated to a square matrix.

5. If A is a matrix of order 3×3 then $|KA|$ is:
(a) $K|A|$ (b) $K^2|A|$ (c) $K^3|A|$ (d) $3K|A|$
Answer: (c) $K^3|A|$

6. If A is non-singular square matrix of order 3×3, then $|adj.A|$ is equal to:
(a) $|A|$ (b) $|A|^2$ (c) $|A|^3$ (d) $3|A|$
Answer: (b) $|A|^2$

Match the columns:

7.

Column-A	Column-B
(i) $\|A\|$	(a) $\|A\| = 0$
(ii) Singular Matrix A	(b) $\|A\| \neq 0$
	(c) $\|A'\|$

(**Answer:-** $\boldsymbol{i.} \rightarrow (\boldsymbol{c})$, $\boldsymbol{ii.} \rightarrow (\boldsymbol{a})$)

8.

Column-A	Column-B
(i) $AB = BA = I$	(a) $A = B = 0$
(ii) $(A^{-1})^{-1}$	(b) $A^{-1} = B \ or \ B^{-1} = A$
	(c) A

(**Answer:-** $\boldsymbol{i.} \rightarrow (\boldsymbol{b})$, $\boldsymbol{ii.} \rightarrow (\boldsymbol{c})$)

9.

Column-A	Column-B
(i) $A(adj\ A)$	(a) A
(ii) AI	(b) $\|A\|I$
	(c) I

(**Answer:-** $\boldsymbol{i.} \rightarrow (\boldsymbol{b})$, $\boldsymbol{ii.} \rightarrow (\boldsymbol{a})$)

10.

Column-A	Column-B
(i) $\|A\| = 0$	(a) $A = \begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}$

(ii) $|A| = 2$ (b) $A = \begin{bmatrix} 3 & 4 \\ -1 & -1 \end{bmatrix}$

(c) $A = \begin{bmatrix} 5 & 4 \\ 2 & 2 \end{bmatrix}$

(**Answer:-** $\boldsymbol{i.} \rightarrow (\boldsymbol{a})$, $\boldsymbol{ii.} \rightarrow (\boldsymbol{c})$)

11.

Column A		Column B	
(i)	$Non - Singular\ matrix\ A$	(a)	$\|A\| = 0$
(ii)	$Singular\ matrix\ A$	(b)	$\|A\| \neq 0$

(**Answer:-** $\boldsymbol{i.} \rightarrow (\boldsymbol{b})$, $\boldsymbol{ii.} \rightarrow (\boldsymbol{a})$)

12.

Column A		Column B	
(i)	$\begin{vmatrix} x & 2 \\ 3 & 4 \end{vmatrix} = 0$	(a)	$x = {}^{4}/_{3}$
(ii)	$\begin{vmatrix} 4 & 3 \\ x & 1 \end{vmatrix} = 0$	(b)	$x = {}^{3}/_{4}$
		(c)	$x = {}^{3}/_{2}$

(**Answer:-** $\boldsymbol{i.} \rightarrow (\boldsymbol{c})$, $\boldsymbol{ii.} \rightarrow (\boldsymbol{a})$)

Fill in the blanks from the following options:

(Square, 10, $9\ |A|$, $27\ |A|$, 25, 125, 0, 1, singular, non-singular)

13. Determinant is a number associated to a ……matrix. **Answer:** Square
14. If $|A| = 10$ then $|A'| =$ …… **Answer:** 10
15. $\begin{vmatrix} x & x+1 \\ x-1 & x \end{vmatrix} =$…… **Answer:** 1
16. If $|A| = 0$ then A is a ……matrix. **Answer:** singular
17. If $|A| \neq 0$ then A is a ……matrix. **Answer:** non-singular
18. If A is a matrix of order 3×3 and $|A| = 5$ then $|adj.\ A| =$…… **Answer:** 25
19. If A is a matrix of order 3×3 then $|3A| =$…… **Answer:** $27\ |A|$

State as true or false:

20. The value of determinant changes if its rows and column are interchanged. (×)
21. If any two rows of a determinant are inter-changed then sign of determinant changes. (✓)
22. If any two rows of a determinant are identical then value of determinant is non-zero. (×)
23. Value of determinant changes when it is expanded by different rows or columns. (×)
24. Area of a triangle cannot be calculated using determinants. (×)
25. A system of linear equations can be solved by matrices and determinants. (✓)
26. Minors and co-factors of determinants are one and the same things. (×)

3 Marks Questions

1. Using determinants find the equation of the line passing from the points (2, -6) and (4, 5).
2. Find the area of triangle with vertices (2, 3), (5,7) and (9, -3).

3. Find the values of K if area of triangle is 4 sq. units and vertices are (K, 0), (4, 3), (5, 4).
4. Find the minor M_{23}, M_{31}, M_{33} in the determinant

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 4 & -1 & 7 \\ 6 & 0 & 8 \end{vmatrix}$$

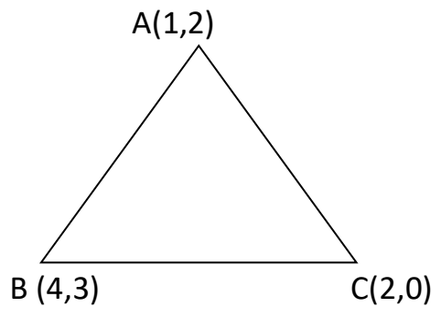
5. Find the co- factors A_{11}, A_{22}, and A_{32} in the determinant

$$\Delta = \begin{vmatrix} 4 & -1 & 0 \\ 3 & 7 & 8 \\ 5 & 3 & 6 \end{vmatrix}$$

6. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then show that $|3A| = 9|A|$
7. Prove that : $\begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} = 0$
8. Prove that: $\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & y^2 \end{vmatrix} = (x-y)(y-z)(z-x)$
9. If $A = \begin{bmatrix} 2 & 6 \\ 5 & 1 \end{bmatrix}$ then find $adj.(A)$
10. Using matrices solve the equations:
$2x + 5y = 1, 3x + 2y = 7$
11. If $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$ then show that $A.(adj.A) = (adj.A)\ A$
12. If $A = \begin{bmatrix} 4 & 5 \\ 2 & 3 \end{bmatrix}$ then find A^{-1}

4 Marks Questions

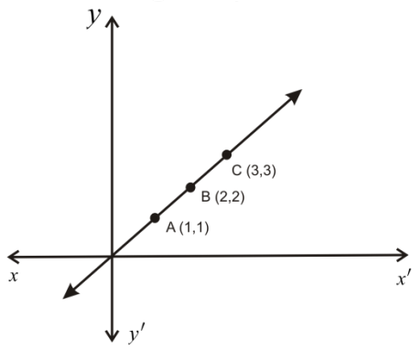
1. Find the area of ΔABC using determinants.



2. Write the differences between matrix and determinant.
3. Write the differences between minors and co-factors.
4. Give two examples of determinants with value zero.
5. Give two examples of square matrices whose determinant is zero.
6. Give two examples of 2×2 non-singular matrices.

7. Compare the values of $\Delta = \begin{vmatrix} 2 & 3 & 1 \\ -1 & 4 & 2 \\ 3 & 0 & 1 \end{vmatrix}$ with the determinant obtained after $R_1 \rightleftharpoons R_2$

8. Using determinant show that given points in the figure are collinear.



9. Give one example each of a singular matrix and non-singular matrix of order 2×2

10. Write the numbers in spaces for which given determinant vanishes.

$$\begin{vmatrix} 0 & 4 & - \\ - & 0 & 6 \\ 5 & -6 & 0 \end{vmatrix}$$

Continuity and Differentiability

Multiple Choice Questions:

1. If $x = at^2, y = 2at$, then $\frac{dy}{dx}$ is :

(a) t (b) 0 (c) $\frac{1}{t}$ (d) a **Ans = (c)**

2. The derivative of $\cos 5x$ w.r.t. x is

(a) $5\sin 5x$ (b) $\sin 5x$ (c) $-5\sin 5x$ (d) $5\cos 5x$ **Ans = (c)**

3. If $f(x) = \begin{cases} kx - 2, x \leq 4 \\ 1 + 2x, x > 4 \end{cases}$ is a continuous function, then the value of k is

(a) $\frac{11}{4}$ (b) $\frac{-5}{4}$ (c) $\frac{7}{4}$ (d) $\frac{4}{11}$ **Ans = (a)**

4. If $x^3 + y^3 = 10$, then the value of $\frac{dy}{dx}$ is

(a) $\frac{-y^2}{x^2}$ (b) $\frac{-x^2}{y^2}$ (c) $\frac{x^2}{y^2}$ (d) $\frac{y^2}{x^2}$ **Ans = (b)**

5. If $y = cos^{-1}\left[\frac{\sqrt{x}-1}{\sqrt{x}+1}\right] + sin^{-1}\left[\frac{\sqrt{x}-1}{\sqrt{x}+1}\right]$ then $\frac{dy}{dx}$ is equal to

(a) 1 (b) $\frac{\sqrt{x}+1}{\sqrt{x}-1}$ (c) $\frac{\sqrt{x}-1}{\sqrt{x}+1}$ (d) 0 **Ans = (d)**

6. The derivative of $\tan\left(\frac{\pi}{2} - x\right)$ is equal to

(a) $\sec^2\left(\frac{\pi}{2} - x\right)$ (b) $-\cos ec^2 x$ (c) $\cos ec^2 x$ (d) $\tan^2\left(\frac{\pi}{2} - x\right)$ Ans = (b)

Match the Column:

7.

	Column - A		Column - B
(a)	$\frac{d}{dx}(e^{-nx})$	(i)	$-ne^{-nx}$
(b)	$\frac{d}{dx}(e^{nx})$	(ii)	$-ne^{nx}$
		(iii)	ne^{nx}

Ans $\begin{bmatrix}(a)\to(i)\\(b)\to(iii)\end{bmatrix}$

8.

	Column -A	Column - B
(a)	$\frac{d}{dx}\tan^{-1}(\cot x)$	(i) 0
(b)	$\frac{d}{dx}(\sec^{-1}x+\text{coses}^{-1}x)$	(ii) 1
		(iii)-1

Ans $\begin{bmatrix}(a)\to(iii)\\(b)\to(i)\end{bmatrix}$

9.

	Column-A		Column-B
(a)	$\frac{d}{dx}(n^x)$	(i)	$x^a \log a$
(b)	$\frac{d}{dx}(x^a)$	(ii)	ax^{a-1}
		(iii)	$n^x \log n$

Ans $(a)\to(iii)$
$(b)\to(ii)$

10.

	Column - A		Column - B
(a)	$\frac{d}{dx}(\sin x)$ at $x=\frac{\pi}{2}$	(i)	0
(b)	$\frac{d}{dx}(\cos x)$ at $x=\frac{\pi}{2}$	(ii)	1
		(iii)	-1

Ans $\begin{bmatrix}(a)\to(i)\\(b)\to(iii)\end{bmatrix}$

11.

	Column-A		Column-B
(a)	$x=2at^2, y=at^4$	(i)	Implicit function
(b)	$x^2+xy+y^2=100$	(ii)	Logarithmic function
		(iii)	Function in Parametric from

Ans $\begin{bmatrix}(a)\to(iii)\\(b)\to(i)\end{bmatrix}$

12.

	Column-A		Column-B
(a)	$\underset{x\to0}{Lt}\frac{\sin 2x}{x}=$	(i)	3
(b)	$\underset{x\to0}{Lt}\frac{\tan 3x}{3x}=$	(ii)	2
		(iii)	1

Ans $\begin{bmatrix}(a)\to(ii)\\(b)\to(iii)\end{bmatrix}$

13.

	Column-A		Column -B
(a)	$\frac{d}{dx}(\sin^{-1}x)$	(i)	$\frac{1}{\sqrt{1-x^2}}$
(b)	$\frac{d}{dx}(\cos^{-1}x)$	(ii)	$\frac{1}{1+x^2}$

(iii) $\frac{-1}{\sqrt{1-x^2}}$ **Ans** $\begin{bmatrix}(a)\to(i)\\(b)\to(iii)\end{bmatrix}$

Fill in the blanks from followings options :

$(\frac{2}{3x}, \frac{-1}{x^2}-2,$ Integral points,applicable, $\frac{\cos(\log x)}{x}, \frac{5}{2}, 3x^2, \frac{\sqrt{3}+1}{2}, \frac{3}{2}, \frac{\sin(\log x)}{x}, \frac{\sqrt{3}-1}{2})$.

14. The derivative of sin (log x) is _____

Ans : $\frac{\cos(\log x)}{x}$

15. If $y = \log x - x^2$ then $y_2 =$ _____

Ans : $\frac{-1}{x^2}-2$

16. The derivative of x^2 w. r. t. x^3 is _____

Ans : $\frac{2}{3x}$

17. The function $f(x)=[x]$ is discontinuous at all _____.

Ans : Integral points.

18. If $f(x)=\sin x-\cos x,$ then $f'(\frac{\pi}{3})$ is equal to _____.

Ans : $\frac{\sqrt{3}+1}{2}$

19. Mean Value theorem for the function
$f(x)=x^2-2x, x\in[1,2]$ is _____.

Ans : applicable

20. The derivative of $e^{3\log x} w.r.t. x$ is equal to _____.

Ans : $3x^2$

State True or False:

21. Trigonometric functions are differentiable functions in their respective domains .

Ans : True

22. $\frac{d}{dx}(e^{\sin^{-1}x}) = e^{\cos^{-1}x}\left(\frac{1}{\sqrt{1-x^2}}\right)$ **Ans : False**

23. If $x=ct, y=\frac{c}{t}$, then $\frac{dy}{dx}$ at t = 2 is 4 .

Ans : False

24. $|x|$ is a continuous function .

Ans : True

25. Every differentiable function is a Continuous functions.

Ans : True

26. $\frac{d}{dx}(tan^{-1}x) = \frac{-1}{1+x^2}$ **Ans : False**

27. The Composition of two continuous function is Continuous.

Ans : True

3 Marks Questions

28. If $y=\frac{1}{x^{19}}$, then find $\frac{dy}{dx}$.

29. If $x=a\sec\theta, y=b\tan\theta$, find $\frac{dy}{dx}$.

30. Differentiate $y = x^x$ w. r. t. x

31. If $2x + 3y = \sin x$, find $\frac{dy}{dx}$.

32. Find $\frac{dy}{dx}$ if $x = a(\theta - \sin\theta), y = a(1 + cos\theta)$,

33. If $y = x\cos x$, then find $\frac{d^2y}{dx^2}$.

34. Find k, if $f(x) = \begin{cases} \frac{x^2 - 9}{x - 3}, x \neq 3 \\ k, \quad x = 3 \end{cases}$ is continuous at $x = 3$.

35. Discuss the continuity of the function

$$f(x) = \begin{cases} \frac{\sin x}{2x}, x \neq 0 \\ \frac{1}{2}, \quad x = 0 \end{cases}$$ at $x = 0$

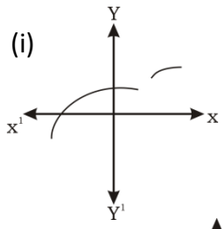
36. Check the applicability of Rolle's Theorem for the function
$f(x) = x^2 + 2x - 8, \ x \in [-4, 2]$

37. Discuss the applicability of Lagrange's Mean Value theorem to $f(x) = x^{3/4}, x \in [-1, 1]$.

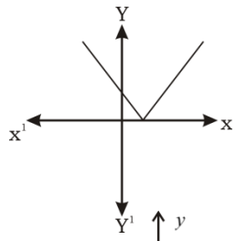
4 Marks Questions :

38. Which of the following graphs are of continuous and discontinuous functions?

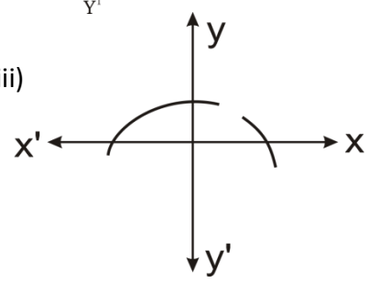
(i)



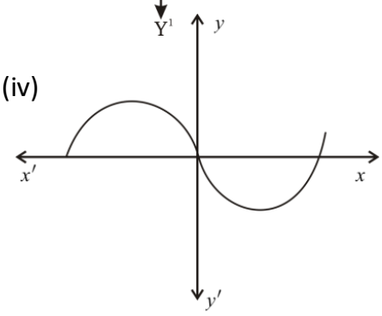
(ii)



(iii)



(iv)



39. Write the formula of differentiation using :

(i) Product rule (ii) Quotient rule

40. Differentiate $y = \frac{e^x + e^{-x}}{e^x - e^{-x}}$ w. r. t . x

41. Write the formula for finding derivative of absolute function $|f(x)|$. Hence find $f'(x)$ if $f(x) = |2x - 3|$.

42. Verify LMV theorem for $f(x) = x^2 - 2x + 4$ in [1,5].

43. Differentiate $y = \sin^{-1}\left(\frac{2x}{1 + x^2}\right), -1 < x < 1$ w. r. t . x

44. If $y = 5\cos x - 3\sin x$, prove that $\frac{d^2y}{dx^2} + y = 0$.

45. If $y = (\sin x)^{\cos x}$ find $\frac{dy}{dx}$.

46. Examine the continuity of

$$f(x) = \begin{cases} \frac{\sin 2x}{\sin 3x} & , \quad x \neq 0 \\ 2 & , \quad x = 0 \end{cases} \qquad \text{at } x = 0$$

Applications of Derivative

Multiple choice Questions:

1. The slope of normal to the curve $y = x^2 + 3$ at $x = 1$ is :

   (a) 2 (b) $\frac{-1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{-1}{2}$ **Ans : (d)**

2. The value of x for which $\cos 2x$ attains its minimum value is :

   (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{6}$ **Ans : (c)**

3. The slope of tangent to the curve $y = 2x^2 + 3\sin x$ at $x = 0$ is :

   (a) 3 (b) -3 (c) 4 (d) -4 **Ans : (a)**

4. The interval in which the function $f(x) = x^2 - 6x + 3$ is strictly increasing is

   (a) (1,+ ∞) (b) (1,2) (c) (3, + ∞) (d) ($-\infty$, 3)

   Ans : (c)

5. The Point where tangent to curve $y = x^2 - 4x + 5$ is parallel to $x - axis$ is :

   (a) (2,1) (b) (1,2) (c) (2,4) (d) (-4,5)

   Ans : (a)

6. The maximum value of $f(x) = x^3 - 3x$ in the interval [0,2] is :

   (a) -2 (b) 0 (c) 2 (d) 1

   Ans : (c)

7. The rate of change of the area of a circle with respect to its radius at r = 5 is :

   (a) 10π (b) 8π (c) 12π (d) 13π

   Ans : (a)

8. The tangent to a given curve is parallel to $x - axis$ if.

(a) $\frac{dy}{dx} = 1$ **(b)** $\frac{dy}{dx} = 0$ **(iii)** $\frac{dy}{dx} = -1$ **(d)** $\frac{dy}{dx} = 2$

Ans : (b)

Match the column:

		Column - A	**Column - B**
9.	(a)	The slope of tangent to curve given by $x = 1 - \cos\theta, y = \theta - \sin\theta$ **at** $\theta = \frac{\pi}{2}$ **is**	(i) $\frac{1}{2}$
	(b)	The slope of tangent to the curve $x = at^2, y = 2at$ at t = 2 is	(ii) 0 (iii) 1

Ans $\begin{bmatrix} (a) \to (iii) \\ (b) \to (i) \end{bmatrix}$

10.		**Column - A**	**Column - B**
	(a)	The slope of tangent to	(i) $\frac{1}{2}$

the curve $y = x^3 - x$ at $x = 2$ (ii) 11

(b) The slope of normal to the Curve $y = 2x^3 - 1$ at $x = 1$ (iii) $\frac{-1}{6}$

Ans $\begin{bmatrix}(a) \to (ii)\\(b) \to (iii)\end{bmatrix}$

11.

	Column - A	**Column - B**
(a)	The minimum Value for the function $f(x) = (2x-1)^2 + 3$ is	(i) 3
(b)	The minimum value for the function $f(x) = 16(x-1)^2 + 24$	(ii) 24
		(iii) 16

Ans $\begin{bmatrix}(a) \to (i)\\(b) \to (ii)\end{bmatrix}$

12.

	Column - A	**Column - B**
(a)	Rate of change of volume of sphere, w.r.t. its radius.	(i) 4
(b)	Rate of change of perimeter of square w.r.t. its side .	(ii) $\frac{4}{3}\pi r^3$
		(iii) $4\pi r^2$

Ans $\begin{bmatrix}(a) \to (iii)\\(b) \to (i)\end{bmatrix}$

13.

	Column - A	**Column - B**
(a)	$f(x) = 3x^2 + 17$	(i) Strictly increasing in $(0, \infty)$
(b)	$f(x) = \lvert x \rvert$	(ii) Strictly increasing on **R**
		(iii) Strictly decreasing in $[-\infty, 0)$

Ans $\begin{bmatrix}(a) \longrightarrow (i)\\(b) \longrightarrow (iii)\end{bmatrix}$

14.

	Column -A	**Column - B**
(a)	The rate of change of area of a circle w.r.t. its radius r is	(i) $2\pi r$
		(ii) $8\pi r$
(b)	The rate of change of surface area of a ball w.r.t. its radius r is	(iii) $\frac{4}{3}\pi r^3$

Ans $\begin{bmatrix}(a) \to (i)\\(b) \to (ii)\end{bmatrix}$

15.

	Column - A	**Column - B**
(a)	The equation of tangent line to the curve $y = x^2$ at (0,0)	(i) $3x - y - 2 = 0$
(b)	The equation of tangent line to the curve $y = x^3$ at (1,1)	(ii) $y = 0$
		(iii) $x = 0$

Ans $\begin{bmatrix}(a) \to (ii)\\(b) \to (i)\end{bmatrix}$

Fill in the blanks from the following options :

(percentage, equilateral, 3, -1, increasing, decreasing, 1, critical point, relative error, isosceles)

16. The Point where $f'(x) = 0$ is called _____.

Ans: critical point

17. If $f'(x) \geq 0$, then the function is _____

Ans: Increasing

18. The local minimum Value of the function is given by $f(x) = 3 + |x|, x \in \mathrm{R}$ _____

Ans: 3

19. The Slope of tangent to the curve $y = \sin x$ at (0,0) is _____.

Ans: 1

20. If two lines are perpendicular then product of their slopes is_____

Ans: - 1

21. The triangle of maximum area that can be inscribed in a given circle is an _____triangle.

Ans: Equilateral

22. $\frac{\Delta x}{x} \times 100$ is called the _____error in x.

Ans: Percentage

State True or False

23. $f(x) = \sin x$ is strictly decreasing function in $(0, \frac{\pi}{2})$

Ans: False

24. If x is real then maximum value of $x^2 - 8x + 17$ is 2 **Ans: False**

25. The equation of tangent to the curve $y = f(x)$ at the given point (x_1, y_1) **is** $y - y_1 = \frac{dy}{dx}(x - x_1)$ **Ans: True**

26. The value of function f is maximum at $x = a$ if $f'(a) = 0$ and $f''(a) < 0$

Ans: True

27. The minimum value for $f(x) = x^2, x \in \mathrm{R}$ is zero

Ans: True

28. The logarithmic functions is strictly increasing on $(0, \infty)$

Ans: True

29. The interval in which $f(x) = 2x^2 - 3x$ is strictly decreasing is $(\frac{3}{4}, \infty)$

Ans: False

4 Marks Questions

30. Find the approximate value of $\sqrt{401}$

31. Find the rate of change of area of a circle w.r.t. its radius r at $r = 6$ cm.

32. Prove that $f(x) = \cos x$ is strictly decreasing on $(0, \pi)$

33. Find the slope of tangent to the curve.

$y = x^3 - x$ at $x = 2$.

34. Find the slope of normal to the curve

$y = x^3 - x$ at $x = 2$

35. Find the points at which tangent to the curve

$y = x^3 - 3x^2 - 9x + 7$ is parallel to $x - axis$.

36. Find all the points of local maxima and local minima of the function f given by :

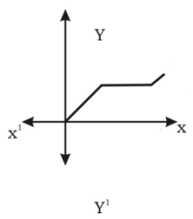
$f(x) = 2x^3 - 6x^2 + 6x + 5$

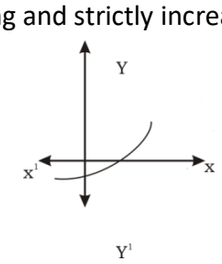
37. Find the approximate value of (i) $\sqrt{37}$ (ii) $\sqrt{49.5}$

38. Find the equation of normal line to the curve

$y = \sin^2 x$ at $x = \frac{\pi}{2}$

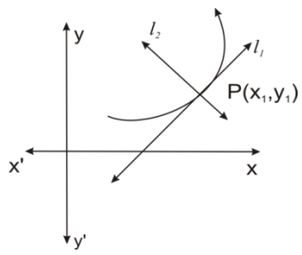
39. Which of the following graphs represents increasing and strictly increasing function.





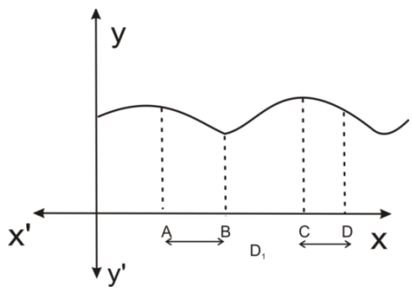
(i) (ii)

40. A curve C is represented in the following figure and two lines l_1, l_2 are drawn. Name the line which is tangent and normal to the curve. Also write equation of tangent and normal line to the curve $y = \text{f}(x)$ at the given point $P(x_1, y_1)$



41. Write the condition of slopes of tangents when two lines are parallel. Hence find the equation of tangent to the curve $x^2 + 3y = 3$ which is parallel to the line $y - 4x + 5 = 0$.

42. Let a real valued function f be defined in the domain of f i.e. D_f,. Write the points from the given figure which describe absolute maxima and absolute minima.



Also find absolute maximum and minimum value if the function $f(x) = x^{50} - x^{20}$ in the interval [0,1]

43. Find two positive numbers whose sum is 24 and their sum of squares is minimum.

44. A square piece of tin of side 24 cm is to be made in to a box without top by cutting a square from each corner and folding up the flaps to form a box. What should be the side of the square to be cut off so that the value of the box is maximum.

45. Draw the rough diagram of the curve $\frac{x^2}{9} + \frac{y^2}{16} = 1$ and draw the tangent lines parallel to $x - axis$ is and $y = axis$ is. Also write their equations from the figure drawn.

46. Using differentials, find the approximate value of $(15)^{1/4}$

47. Let $f(x)$ be continuous on [a,b] and differentiable on (a,b). Write the conditions on derivative of $f(x)$ when

(i) $f(x)$ is increasing in [a,b]

(ii) $f(x)$ is decreasing in (a,b]

INTEGRALS

Multiple Choice Questions:

1. If $\int f(x)dx = g(x) + c$, then

(a) $g(x) = f(x)$ (b) $\frac{d}{dx} g(x) = f(x)$

(c) $\frac{d}{dx} f(x) = g(x)$ (d) $g(x) \neq f(x)$

Ans : (b)

2. $\int x^{-1} dx =$ .....

(a) $\frac{x^0}{c} + c$ (b) $\log_e x + c$

(c) $\log_{10} x + c$ (d) $\log_e |x| + c$

Ans : (d)

3. Which of the following is equal to $\int \frac{dx}{\sqrt{1-x^2}}$

(a) $sin^{-1}x + c$ (b) $cos^{-1}x + c$

(c) $\frac{\pi}{2} + cos^{-1}x + c$ (d) $tan^{-1}x + c$ **Ans : (a)**

4. $\int e^x[f(x) + f'(x)]dx$ is equal to

(a) $e^x f'(x) + c$ (b) $e^x f(x) + c$

(c) $e^x + f(x) + c$ (d) $e^x - f(x) + c$ **Ans : (b)**

5. $\int e^x(\cos x - \sin x)dx$ is equal to

(a) $e^x \sin x + c$ (b) $-e^x \cos\ x + c$

(c) $e^x \cos x + c$ (d) $-e^x \sin x + c$ **Ans : (c)**

6. $\int \tan x \sec^2 x$ is equal to

(a) $\tan x + c$ (b) $\frac{1}{2}\tan^2 x + c$

(c) $\sec^2 x + c$ (d) $\sec x \tan x + c$ **Ans : (b)**

7. $\int_0^2 [x]dx$ is equal to

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) 0 Ans : (b)

Match the Column

8.

	Column - I		**Column - II**
(a)	$\int \frac{dx}{1+x^2}$	(i)	$\cos ec^{-1}x + c$
(b)	$\int \frac{dx}{\sqrt{1-x^2}}$	(ii)	$\sin^{-1} x + c$
		(iii)	$\tan^{-1} x + c$

Ans $\begin{bmatrix}(a) \to (iii)\\(b) \to (ii)\end{bmatrix}$

9.

	Column - I		**Column - II**
(a)	$\int_{-\pi/2}^{\pi/2} \sin^7 x dx$	(i)	0
(b)	$\int_0^{\pi/2} \frac{\cos^5 x}{\sin^5 x + \cos^5 x} dx$	(ii)	$\frac{\pi}{2}$
		(iii)	$\frac{\pi}{4}$

Ans $\begin{bmatrix}(a) \to (i)\\(b) \to (iii)\end{bmatrix}$

10.

	Column - I		**Column - II**
(a)	$\int_{-a}^{a} f(x)dx = 0$	(i)	If $f(x) = x^3 + 1$
(b)	$\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$	(ii)	If $f(x)$ is an odd function
		(iii)	If $f(x)$ is an even function

Ans $\begin{bmatrix}(a) \to (ii)\\(b) \to (iii)\end{bmatrix}$

11.

	Column - I		**Column - II**
(a)	$\int e^x(\tan^{-1} x + \frac{1}{1+x^2})dx$	(i)	$e^x \sin^{-1} x + c$

(b) $\int e^x(\sin^{-1}x+\frac{1}{\sqrt{1-x^2}})dx$ (ii) $e^x\tan^{-1}x+c$

(iii) $e^x co\sec^{-1}x+c$ **Ans** $\begin{bmatrix}(a)\to(ii)\\(b)\to(i)\end{bmatrix}$

12.

	Column - I		**Column - II**
(a)	$\int\sqrt{a^2-x^2}dx$ =	(i)	$\frac{x\sqrt{a^2-x^2}}{2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+c$
(b)	$\int\sqrt{a^2+x^2}dx=$	(ii)	$\frac{x\sqrt{a^2+x^2}}{2}+\frac{a^2}{2}\log\left(x+\sqrt{a^2+x^2}\right)+c$
		(iii)	$\frac{x\sqrt{x^2-a^2}}{2}-\frac{a^2}{2}\log\left\lvert x+\sqrt{a^2+x^2}\right\rvert+c$

Ans $\begin{bmatrix}(a)\to(i)\\(b)\to(ii)\end{bmatrix}$

13.

	Column - I		**Column - II**
(a)	$\int\frac{ax+b}{(x-p)(x-q)}dx$	(i)	$\frac{A}{x-p}+\frac{B}{(x-p)^2}+\frac{C}{x-q}$
(b)	$\int\frac{ax^2+bx+c}{(x-p)^2(x-q)}dx$	(ii)	$\frac{A}{x-p}+\frac{B}{(x-p)^2}+\frac{C}{x-r}$
		(iii)	$\frac{A}{x-p}+\frac{B}{x-q}$

Ans $\begin{bmatrix}(a)\to(iii)\\(b)\to(ii)\end{bmatrix}$

14.

	Column - I		**Column - II**
(a)	$\int\frac{1}{x^2+2x+3}\,dx$	(i)	$\int\frac{1}{(x+1)^2+2}\,dx$
(b)	$\int\frac{1}{x^2+4x+6}\,dx$	(ii)	$\int\frac{1}{(x-1)^2+2}\,dx$
		(iii)	$\int\frac{1}{(x+2)^2+2}\,dx$

Ans $\begin{bmatrix}(a)\to(i)\\(b)\to(iii)\end{bmatrix}$

Fill in the blanks from the following options:

$\left(\frac{-1}{2x^2},0,1,\frac{\pi}{4},\text{odd, even, }\sec x+\mathbf{c},\ \frac{\pi}{6},-cosec\,x+c,\frac{a^x}{\log a}+c,e^x+c\right)$

15. $\int_0^{\pi/2}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx=$ _____ **Ans :** $\frac{\pi}{4}$

16. $\int_{-a}^{a}f(x)dx=0$ if f is an _____function. **Ans :odd**

17. $\int_{-\pi}^{\pi}\sin^3\cos^2 x dx$ is equal to _____ **Ans :0**

18. $\int e^{-3\log x}dx$ is equal to _____ **Ans :** $-\frac{1}{2x^2}$

19. $\int\frac{\sin x}{\cos^2 x}dx$ is equal to _____ **Ans :** $\sec x+\mathbf{c}$

20. $\int_0^1\frac{dx}{1+x^2}$ is equal to _____ **Ans :** $\frac{\pi}{4}$

21. $\int\cos ecx\cot x\ dx$ is equal to _____ **Ans:** $-cosec\,x+c$

22. $\int a^x dx=$ _____ **Ans :** $\frac{a^x}{\log a}+c$

State True or False:

23. $\int x^n dx = \frac{x^{n+1}}{n+1} + c$ **Ans : True**

24. If $f(a-x) = f(x)$, then $\int_0^{2a} f(x)dx = 0$ **Ans : False**

25. $\int_a^b f(x)dx \neq \int_a^b f(a+b-x)dx$ **Ans : False**

26. $\int_a^b f(x)dx$ if it exists, is a uniquely determined real number. **Ans : True**

27. $\int_0^{2\pi} \sin^2 x \, dx = 4 \int_0^{\pi/2} \sin^2 x \, dx$ **Ans : True**

28. $\int_a^b f(x)dx = \int_b^a f(x)dx$ **Ans : False**

29. $\int_a^b f(x)dx = \lim_{h\to 0} h \int_b^a f(a) + f(a+h) \ldots\ldots + f(a+\overline{(n-1)}\, h$;where *nh = b-a*

Ans : True

3 Marks Questions :

30. Evaluate $\int \frac{dx}{\sqrt{9-25x^2}}$

31. Evaluate $\int \tan^2 x \, dx$

32. Evaluate $\int \frac{1-\tan x}{1+\tan x} dx$

33. Evaluate $\int x \, e^{3x} dx$

34. Evaluate $\int e^x (\sec x + \sec x \tan x) dx.$

35. Evaluate $\int \frac{dx}{x^2+2x+7}$

36. Evaluate $\int_{-1}^{3} (x^2+1)dx$

37. Evaluate $\int_0^1 \frac{x^8}{1+x^9} dx$

38. Evaluate $\int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx$

39. Evaluate $\int_{-2}^{2} |x+1| \, dx$

4 Marks Questions

40. Give any two examples of an intergral which can be solved by using partial fractions.

41. Complete perfect square of the quadratic equation $x^2 + 4x + 5 = 0$.Hence Evaluate $\int \frac{dx}{x^2+4x+5}$.

42. Write the properties to evaluate definite integrals when (i) $f(x)$ is an odd function (ii) $f(x)$ is an even function.

43. Give the formula of definite integral as the limit of a Sum and write *nh* for $= \int_0^1 x \, dx$.

44. Write the name of the rule for integrating the product of two functions. Also Evaluate $\int x \sin x \, dx$.

45. Evaluate $\int \frac{x^2}{x-1} dx$.

46. Evaluate $\int_0^1 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}} dx$

47. Evaluate $\int \frac{dx}{1+\sin x}$

Application of Integrals

Multiple Choice Questions:

1.Using integration, area of circle $x^2 + y^2 = 25$ is:

(a) 5π sq.units (b) 10 sq.units (c) 25π sq.units (d) 10π sq.units

Ans:(c) 25π sq.units

2.Using integration, area of ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$ is:

(a) 20π sq.units (b) 20 sq.units (c) 25π sq.units (d) 16π sq.units

Ans:(a) 20π sq.units

3. Integral for the area of circle $x^2 + y^2 = 16$ is:

(a) $\int_0^4 \sqrt{16-x^2}\ dx$ (b) $\int_0^4 (16-x^2)\ dx$ (c) $4\int_0^{16} \sqrt{16-x^2}\ dx$ (d) $4\int_0^4 \sqrt{16-x^2}\ dx$

Ans: (d) $4\int_0^4 \sqrt{16-x^2}\ dx$

4. Integral for the area bouned by parabola $y^2 = 4x$ and straight lines $x = 1,\ x = 5$ in the first quadrant is:

(a) $\int_1^5 2\sqrt{x}\, dx$ (b) $\int_1^5 4x\, dx$ (c) $\int_1^5 2x\, dx4$ (d) $\int_1^5 16x^2\, dx$

Ans:(a) $\int_1^5 2\sqrt{x}\, dx$

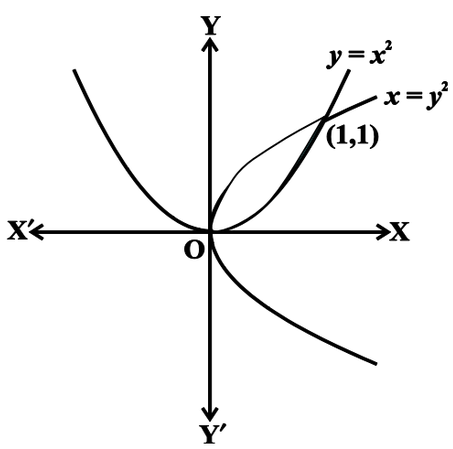
State True or False:

1. We can find the area between curve and x-axis using definite integrals. (T)
2. We can find the area between curves using differentiation. (F)
3. Finding area under the curve is an application of integrals. (T)

3 Marks Questions :

1. Find the area of the region bounded by the curves :
$y = x^2, x = 1, x = 5$ and $x - axis$ [Ans : 41.33]

2. Shade the common region bounded by the parabolas :
$y = x^2$ and $x = y^2$



[Ans : $\frac{1}{3}$]

3. Find the area between the curve :

$y = x^2, x-axis$ and the line $x = 0$ and $x = 2$ [Ans : $\frac{8}{3}$]

4. Formulate the integral for the curve $y = f(x)$ above the $x-axis$, between $x = a$ and $x = b$.

[Ans : $\int_a^b y\,dx$]

5. Using integration, find the area in the first quadrant of circle $x^2 + y^2 = 4$

[Ans : π]

6. Find the area of the region bounded by the curve $y^2 = 4x$ and the line $x = 3$.

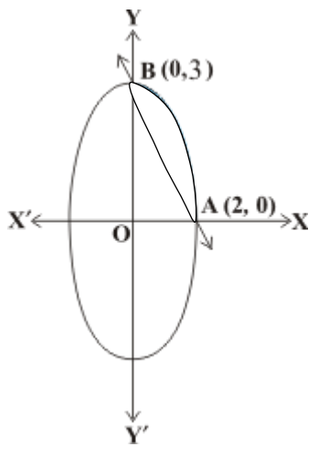
[Ans : $4\sqrt{3}$]

7. Draw the rough sketch of the region bounded by the lines given by : $|x| + |y| = 1$

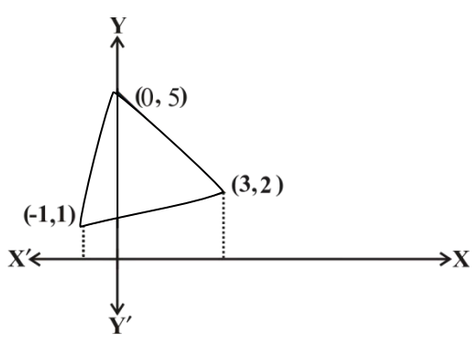
8. Write the formula to evaluate area bounded under two curves $y = f(x)$ and $y = g(x)$ and line $x = a$ and $x = b$ by using integration,where $f(x) > g(x)\ \forall\ x \in [a, b]$ [Ans : Area = $\int_c^b [f(x) - g(x)]dx$,

4 Marks Questions :

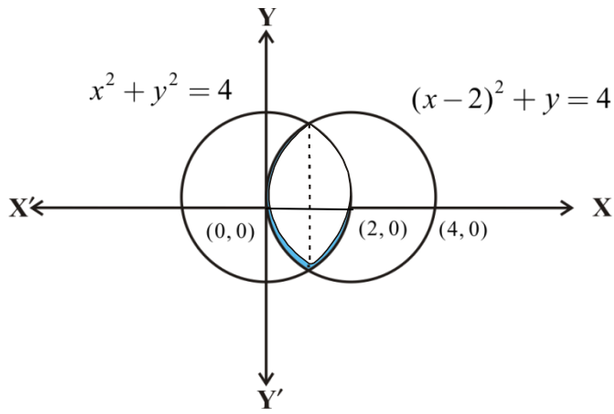
9. In the given figure shade the shorter (smaller) region bounded by the line $\frac{x}{2} + \frac{y}{3} = 1$ and ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$. Also form the integral to evaluate the area of shaded region**.**



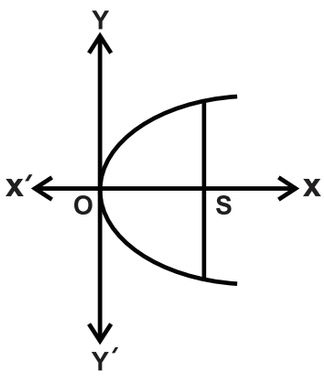
10. In the given figure, shade the region bounded by the vertices (-1,1), (0,5) and (3,2).



11. Shade the region enclosed between the circles $x^2 + y^2 = 4$ and $(x-2)^2 + y^2 = 4$ in the given figure.



12. Find the area bounded by the curves $y^2 = 4x$ and $y = 2x$
13. Find the area bounded by the ellipse :
$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$
14. Find the area enclosed under the parabola $y^2 = 49x$ and its latus - rectum.



15. Draw the rough sketch of two parabolas $y = x^2$ and $x = y^2$ and shade the region enclosed by the curves :
16. Draw the rough sketch of the region of a triangle whose vertices are (2,-2), (4,3) and (1,2).

Differential Equations

Multiple Choice Questions:

1. The order of the given differential equation is :
$$(y''')^2 + (y'')^3 + (y')^4 + y^5 = 0$$
(A) 0 (B) 3 (C) 2 (D) Not defined
[Ans : B]
2. The degree of the given differential equation :
$\frac{dy}{dx} + 5y = 0$ is :
(A) 3 (B) 0 (C) 1 (D) 2 [Ans : C]
3. Solution of the given differential equation.
$\frac{dy}{dx} = \tan^2 x$ is :
(A) $y = \cot - x + c$ (B) $y = \sec x - x + c$
(C) $y = \tan x - x + c$ (D) None of these
[Ans : C]
4. The number of arbitrary constants in the particular solution of a differential equation of fourth order are :
(A) 0 (B) 2 (C) 3 (D) 4
[Ans : A]

5. The number of arbitrary constants in the general solution of a differential equation of third order are .

(A) 3 (B) 2 (C) 1 (D) 0

[Ans : A]

6. Which of the following of differential equation has $y = c_1e^x + c_2e^{-x}$ as the general solution?

(A) $\frac{d^2y}{dx^2} + y = 0$ (B) $\frac{d^2y}{dx^2} - y = 0$

(C) $\frac{d^2y}{dx^2} + 1 = 0$ (D) $\frac{d^2y}{dx} - 1 = 0$ [Ans : B]

7. Which of the following differential equation has $y = x$ as one of its particular solution?

(A) $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = x$ (B) $\frac{d^2y}{dx^2} - x\frac{dy}{dx} + xy = x$

(C) $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = 0$ (D) $\frac{d^2y}{dx^2} - x\frac{dy}{dx} + xy = 3$

[Ans : C]

8. The general solution of differential equation $\frac{dy}{dx} = e^{x+y}$ is;

(A) $e^x + e^{-y} = c$ (B) $e^x + e^y = c$

(C) $e^{-x} + e^y = c$ (D) $e^{-x} + e^{-y} = c$ [Ans : A]

9. A homogenous differential equation of the form $\frac{dx}{dy} = h(\frac{x}{y})$ can be solved by making the substitution :

(A) $y = vx$ (B) $v = yx$ (C) $x = vy$ (D) $x = v$

[Ans : C]

10. Which of the following is homogenous differential equation.

(A) $(4x + 6y + 5)dy - (3y + 2x + 4)dx = 0$

(B) $(xy)dx - (x^3 + y^3)dy = 0$

(C) $(x^3 + 2y^2)dx + 2xy\,dy = 0$

(D) $y^2dx + (x^2 - xy - y^2)dy = 0$ [Ans : D]

Fill in the blanks from the following options :

[$\frac{1}{\sqrt{1-y^2}}$, Positive, $\frac{1}{x}$, mx, Negative]

11. Order and degree (if defined) of a differential equation are always _____integers.

[Ans : Positive]

12. The integrating factor of the differential equation $(1 - y^2)\frac{dx}{dy} + yx = ay, (-1 < y < 1)$ is _____.

[Ans : $\frac{1}{\sqrt{1-y^2}}$]

13. The integrating factor of Differential equation $x\frac{dy}{dx} - y = 2x^2$ is_____.

[Ans : $\frac{1}{x}$]

State as True/False:

15. The given differential equation $(x - y)\frac{dy}{dx} = x + 2y$ is homogenous. [Ans:T]

16. $\frac{dy}{dx} + Py = Q$ is a homogenous differential equation. [Ans : F]

Match the Columns:

17.

Column A		Column B	
(a)	$\frac{dy}{dx} = e^x + 1$	(i)	$e^x + e^{-y} = c$
(b)	$\frac{dy}{dx} = e^{x+y}$	(ii)	$e^x e^y = c$
		(iii)	$y = x + e^x + c$

[Ans : a - (iii), b - (i)]

18.

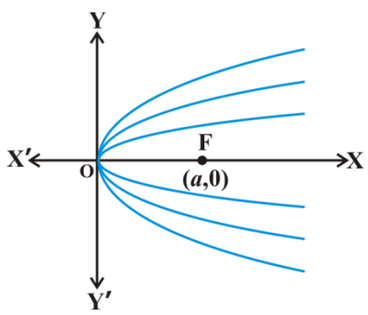
Column A		Column B	
(a)	$\frac{d^2y}{dx^2}$	(i)	y'
(b)	$\frac{d^3y}{dx^3}$	(ii)	y'''
(c)	$\frac{dy}{dx}$	(iii)	y_n
		(iv)	y''

((a)-(iv),(b)-(ii),(c-i))

4 Marks Questions :

19. Give an example of a differential equation whose degree is not defined?

20. Write the differential equation representing the family of parabolas $y^2=4ax$ having vertex at origin and *axis* along positive direction of $x-axis$.

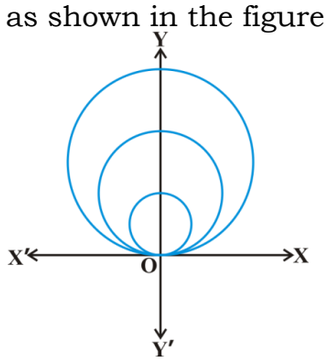


21. Give an example of linear differential equation.

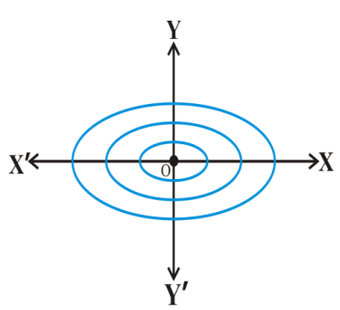
22. Find the integrating factor of differential equation $\frac{dy}{dx} + y = \cos x$

23. Find the general solution of the differential equation
$\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

24. Write the differential equation of family of circles, touching the $x-axis$ at the origin as shown in the figure given below :



25. Write the differential equation of the following family of ellipses, having foci on $x-axis$ and centre at origin (figure given)



Vectors

Choose the correct option from the given options :

1. The unit vector in the direction of the vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$

(a) $\hat{i} + \hat{j} + 2\hat{k}$ (b) $\frac{\hat{i}}{2} + \frac{\hat{j}}{2} + \frac{2\hat{k}}{3}$

(c) $\frac{\hat{i}}{\sqrt{6}} + \frac{\hat{j}}{\sqrt{6}} + \frac{2\hat{k}}{\sqrt{6}}$ (d) $\frac{\hat{i}}{\sqrt{3}} + \frac{\hat{j}}{\sqrt{3}} + \frac{2\hat{k}}{\sqrt{3}}$

[Ans :(c)]

2. If $\vec{a}$ and $\vec{b}$ are two vectors, then scalar projection of vector $\vec{a}$ on vector $\vec{b}$ is :

(a) $\frac{\vec{a} \times \vec{b}}{|\vec{a}|}$ (b) $\frac{\vec{a}.\vec{b}}{|\vec{a}|}$

(c) $\frac{\vec{a} \times \vec{b}}{|\vec{b}|}$ (d) $\frac{\vec{a}.\vec{b}}{|\vec{b}|}$ [Ans :(d)]

3. Area of a parallogram, whose two adjacent sides are given by the two vectors $\vec{a}$ and $\vec{b}$, is :

(a) $|\vec{a} \times \vec{b}|$ (b) $|\vec{a}|$

(c) $|\vec{b}|$ (d) $\frac{1}{2}|\vec{a} \times \vec{b}|$ [Ans :(a)]

4. Area of a parallogram whose diagonals are represent by the two vectors $\vec{d}_1$ and $\vec{d}_2$ is :

(a) $|\vec{d}_1 \times \vec{d}_2|$ (b) $|d_1 . d_2|$

(c) $\frac{1}{2}|\vec{d}_1 \times \vec{d}_2|$ (d) $\frac{1}{2}|d_1 . d_2|$ [Ans :(c)]

Fill in the blanks from the following options :

($\mathbf{0^o}, \vec{a} - \vec{b}, \vec{0}, 1, |\vec{a}||\vec{b}| \cos\theta, |\vec{a}||\vec{b}|$)

5. $\vec{a} + (-\vec{b}) =$ ------ $[Ans.\ \vec{a} - \vec{b}]$

6. $\vec{a} \cdot \vec{b} =$ --------- $[Ans.\ |\vec{a}|\ |\vec{b}|\ \cos\theta]$

7. When two vectors $\vec{a}$ and $\vec{b}$ are parallel to each other then angle between them is ---------- $[Ans.\ 0^0]$

8. $\hat{i} \times i = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} =$ ------ (Ans.$\vec{0}$)

9. $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} =$ ------- $[Ans.\ 1]$

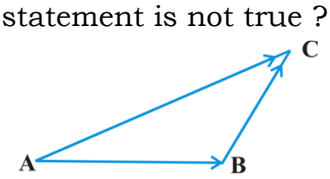
10. For Δ ABC (figure), which of the following statement is not true ?

(A) $\overrightarrow{AB} - \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$

(B) $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \vec{0}$

(C) $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{CA} = \vec{0}$

(D) $\overrightarrow{AB} - \overrightarrow{CB} - \overrightarrow{CA} = \vec{0}$



[Ans :(b)]

11. Which of the following statements is not true?

(a) $(\hat{i} \times \hat{j}) \cdot \hat{k} + \hat{i} \cdot \hat{j} = 1$

(b) $(\hat{k} \times \hat{j}) \cdot \hat{i} + \hat{j} \cdot \hat{k} = 1$

(c) $(\hat{k} \times \hat{i}) \cdot \hat{j} + \hat{i} \cdot \hat{k} = 1$ [Ans :(b)]

3 Marks Questions :

12. Find the sum of vectors :

$\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = -2\hat{i} - 4\hat{j} + 5\hat{k}$, $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$

[Ans : $-12\,\hat{j} - \hat{k}$]

13. If $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$, then find $|\vec{a}|$. [Ans : 3]

14. Find the vector joining the points P (2,3,0) and Q (-1, -2, -4), directed from P to Q.

[Ans : $-3\hat{i} - 5\hat{j} - 4\hat{k}$]

15. Show that the vectors $2\hat{i} + 3\hat{j} - 4\hat{k}$ and $4\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear.

16. Find the unit vector in the direction of the vector $\vec{a} = -\hat{i} + 2\hat{j} + 2\hat{k}$.

[Ans : $\frac{-1}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$]

17. Find the direction cosine of the vector given by $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

Ans : $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}\frac{3}{\sqrt{14}}$]

18. Find the value of 'x' and 'y' if the two vectors $2\hat{i} + 3\hat{j}$ and $x\hat{i} + y\hat{j}$ are equal.

[Ans : $x = 2, y = 3$]

19. Find the angle between vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$.

[Ans : $\cos^{-1}(\frac{5}{7})$]

20. Show that the vectors $\vec{a} = 2\hat{i} + 3\hat{j}$ and $\vec{b} = 4\hat{i} + 6\hat{j}$ are parallel.

21. Find $|\vec{a} \times \vec{b}|$ if $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 5\hat{j} + 2\hat{k}$.

[Ans : $\sqrt{243}$]

22. Find the area of a parallogram if two adjacent sides of a parallogram are $\vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$.

[Ans : $\sqrt{42}$]

23. Find the value of : $(3\vec{a} - 5\vec{b}).(2\vec{a} + 7\vec{b})$.

24. Find the value of 'p' for which the vectors :
$3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\hat{i} - 2p\hat{j} + 3\hat{k}$ are parallel. [Ans : $p = (-\frac{1}{3})$]

Three Dimensional Geometry

Fill in the blanks from following options:

$(1, a^1a^2 + b^1b^2 + c^1c^2 = 0, (\overrightarrow{a_2} - \overrightarrow{a_1}).(\overrightarrow{b_1} \times \overrightarrow{b_2}) = 0, \vec{r} = -\hat{i} + 2\hat{k} \pm \lambda(3\hat{i} + 4\hat{j} + 6\hat{k}),$
$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1, \left|\frac{(\overrightarrow{b_1} \times \overrightarrow{b_2}).(\overrightarrow{a_2} - \overrightarrow{a_1})}{|(\overrightarrow{b_1} \times \overrightarrow{b_2})|}\right|, < \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}} >)$

1. If <l,m,n> are direction cosines of a line, then $l^2+m^2+n^2$ = _____. Answer :1

2. The vector equation of a line passing through the points (-1,0,2) and (3,4,6)
is _____. Answer : $\bar{r} = -\hat{i} + 2\hat{k} + \lambda(\hat{i} + \hat{j} + \hat{k})$

3. The shortest distance between the lines $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ *and* $\vec{r} = \overrightarrow{a_2} + \mu\overrightarrow{b_2}$
is _____.

Answer : $\left|\frac{(\overrightarrow{b_1} \times \overrightarrow{b_2}).(\overrightarrow{a_2} - \overrightarrow{a_1})}{|(\overrightarrow{b_1} \times \overrightarrow{b_2})|}\right|$

4. Direction cosines of a line which makes equal angles with co-ordinate axes are _____. Answer : $< \frac{\pm 1}{\sqrt{3}}, \frac{\pm 1}{\sqrt{3}}, \frac{\pm 1}{\sqrt{3}}, >$

5. Two lines with direction ratios a_1,b_1,c_1 and a_2,b_2,c_2 are perpendicular if _____.

   Answer : $a_1a_2 + b_1b_2 + c_1c_2 = 0$

6. Intercept form of the equation of a plane is _____.

   Answer : $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

7. Two lines $r = a_1 + \lambda b_1$ and $r = a_2 + \mu b_2$ are coplanar if _____.

   Answer : $(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2) = 0$

Multiple Choice Questions:

8. The direction cosines to the normal to the plane 2x+3y-z = 5 are

   (a) <2,3,-1> (b) <-2,-3,1> (c) $<\frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{-1}{\sqrt{14}}>$ (d) $<\frac{2}{5}, \frac{3}{5}, \frac{-1}{5}>$

   Answer : (c)

9. The direction ratios of a line joining the points A(2,3,-4) and B(1-2,3) are

   (a) <1,5,7> (b) <1,-5,7> (c) $|1, -5, -7|$ (d) < 1,5, -7 >

   Answer : (d)

10. The equation of plane passing through origin is

    (a) ax+by+cz = 3 (b) ax+by+cz = 1 (c) ax+by+cz = 0 (d) ax+by+cz = $\sqrt{3}$

    Answer : **(c)**

11. The lines $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ are

    (a) Perpendicular (b) Parallel
    (c) Intersecting (d) None of these

    Answer : (a)

12. Angle between two planes $\vec{r}.\hat{n}_1 = d_1$ and $\vec{r}.\hat{n}_2 = d_2$ is

    (a) $\cos\theta = \left|\frac{\hat{n}_1 - \hat{n}_2}{|\hat{n}_1||\hat{n}_2|}\right|$ (c) $\sin\theta = \left|\frac{\hat{n}_1.\hat{n}_2}{|\hat{n}_1||\hat{n}_2|}\right|$

    (b) $\cos\theta = \left|\frac{\hat{n}_1.\hat{n}_1}{|\hat{n}_1||\hat{n}_1|}\right|$ (d) $\sin\theta = \left|\frac{\hat{n}_1 - \hat{n}_2}{|\hat{n}_1||\hat{n}_2|}\right|$

    Answer : (b)

13. The intercepts cut off by the plane 2x+y-z = 5 are

    (a) $\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}$ (b) 2,1, -1 (c) $\frac{2}{5}, \frac{1}{5}, \frac{-1}{5}$ (d) $\frac{5}{2}, 5, -5$ Answer : (d)

State as True/False:

14. Direction Cosines of XY plane are <1,1,0> (F)

15. The pair of lines given by $\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(6\hat{i} + 4\hat{j} - 8\hat{k})$ and $\vec{r} = -5\hat{i} + 7\hat{j} - 4\hat{k} + \mu(3\hat{i} + 2\hat{j} - 4\hat{k})$ are parallel . (T)

16. The equation of line passing through the point (1,2,3) and parallel to the vector $3\hat{i} + 2\hat{j} - 4\hat{k}$ is $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \mu(3\hat{i} + 2\hat{j} - 4\hat{k})$ (F)

17. The equation of plane with direction ratios a,b,c is ax+by+cz = 0 (F)

18. The angle between the lines passing through origin and direction ratios a_1,b_1,c_1 and a_2,b_2,c_2 is $\sin\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{{a_1}^2 + {b_1}^2 + {c_1}^2}\sqrt{{a_2}^2 + {b_2}^2 + {c_2}^2}}$ (F)

19. The equation of a line passing through two points (x_1,y_1,z_1) and (x_2,y_2,z_2) is $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$ (F)

Match the following:

20. (i) Lines are perpendicular
(ii) Lines are parallel

(p) $l^2+m^2+n^2 = 1$
(q) $a_1a_2 + b_1b_2 + c_1c_2 = 0$
(r) $a_1 = ka_2$, $b_1 = kb_2$, $c_1 = kc_2$
Answer (i) - (q), (ii)- (r)

21. (i) Equation of a line passing through one point and parallel to given vector
(ii) Equation of plane in normal form

(p) $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$
(q) $\vec{r}.\hat{n} = d$
(r) $\vec{r} = \vec{a} + \lambda\vec{b}$
Answer (i) - (r), (ii)- (q)

22. (i) Shortest distance between two lines
(ii) Distance of origin from plane $\vec{r}.\hat{n} = d$

(p) d
(q $\left|\frac{\hat{b} \times (\vec{a}_2 - \vec{a}_1)}{|b|}\right|$
(r) $\left|\frac{(\vec{a}_2 - \vec{a}_1).(\vec{b}_2 - \vec{b}_1)}{|\vec{b}_1 \times \vec{b}_2|}\right|$
Answer (i) - (r), (ii) -(p)

23. (i) Two lines are coplanar
(ii)Equation of plane through three non collinear points

(p) $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$

(q) $a.(x-x_1) + b.(y-y_1) + c.(z-z_1) = 0$
(r)
$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$
Answer (i) - (r), (ii) -(p)

3 Marks Questions

24. Find the angle between the pair of lines given by $\vec{r} = 3\hat{\imath}+2\hat{\jmath}+4\hat{k} + \lambda(\hat{\imath}+2\hat{\jmath}+2\hat{k})$ and

$\vec{r} = \hat{\imath} + \hat{\jmath} + \hat{k} + \mu\left(3\hat{\imath} + \hat{\jmath} + 2\hat{k}\right)$

25. Find the distance of a point (2,5,-3) from the plane $\vec{r}.(6\hat{\imath}-3\hat{\jmath}+2\hat{k}) = 4$

26. Find direction cosines of the unit vector perpendicular to the plane $\vec{r}.(6\hat{\imath}-3\hat{\jmath}-2\hat{k})+1 = 0$ passing through the origin.

27. Find the value of p so that the lines $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ and $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ are at right angles.

28. Show that the points A(2,3,-4),B(1,-2,3) and (3,8,-11) are collinear.

29. Show that the lines $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$ and $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$ are coplaner.

30. Find the shortest distance between the lines

$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$

31. Find the shortest distance between the lines
$r = \hat{\imath} + 2\hat{\jmath} + \hat{k} + \lambda(\hat{\imath} - \hat{\jmath} + \hat{k})$ $r = 2\hat{\imath} + 2\hat{\jmath} - \hat{k} + \mu(2\hat{\imath} + \hat{\jmath} + 2\hat{k})$

32. Find the equation of a plane through the intersection of planes x+y+z =1 and 2x+3y+4z = 5 which is perpendicular to the plane x-y+z = 0

33. Find the equation of a plane that passes through the points (1,1,0), (1,2,1) and (-2,2,-1)

34. Find the co-ordinates of the foot of perpendicular drawn from the origin to the plane

2x-3y+4z-6 = 0

PROBABILITY

Fill in the blanks from following options:

(5/6 ½,$\Sigma x_i^2 p(x_i)$, impossible , 0.35, 0, not defined, sure, 0.65, 16/25)

1. If P (A) = 0.35, then P($\overline{A}$) _____. **Answer :** 0.65
2. P(A) = 1 is called _____ event. **Answer :** Sure
3. If a dice is tossed once, the probability of getting an even number is _____.
   Answer : $\frac{1}{2}$
4. If P (A) = ½, P(B) = 0 then P(A/B) is _____. **Answer:Not defined**
5. If X is a random variable & Var. (X) = $\frac{25}{36}$ then standard deviation =_____
   Answer : $\frac{5}{6}$
6. Variance of a random variable is $E(X^2) - [E(X)]^2$ where $E(X^2)$ = _____.
   Answer : $\Sigma X_i^2 p(x_i)$
7. If P (A) =0.6, P(B) = 0.5 and P(A∩B) =0.32 then P(A/B) = _____.
   Answer : $\frac{16}{25}$
8. If A is an impossible event than P (A) =_____ **Answer :** 0

Multiple Choice Questions

9. Two cards are drawn from a well shuffled deck of 52 cards with replacement. The probability that both cards are queen is

   (a) 1/13 × 1/13 (b) 1/13 + 1/13 (c) 1/52 × 1/52 (d) 1/52 + 1/52

   Answer :(a)

10. If P(A) =3/5, P(B) = 1/5 and A and B are independent events the P (A and B) is
    (a) 1/3 (b) 25/3 (c) 1/12 (d) 3/25
    Answer : (d)
11. If A and B are events such that P(A/B) = P(B/A) then
    (a) A⊂B but A≠B (b) A = B (c) P(A∩B) = ϕ (d) P(A) = P(B)
    Answer : (d)
12. If P(A) = 6/11, P(B) = 5/11 and P(A∩B) =4/11 then P(A/B) is
    (a) 7/11 (b) 2/5 (c) 3/11 (d) 4/5
    Answer : (d)
13. A family has two children. The probability that both the children are boys given that at least one of them is a boy is
    (a) $\frac{3}{4}$ (b) $\frac{1}{3}$ c) $\frac{1}{4}$ (d) $\frac{2}{3}$
    Answer : (d) $\frac{1}{3}$
14. A random variable X has following probability distribution.

X	0	1	2	3	4
P(X)	K	2K	2K	2K	K

    Then the value of K is
    (a) 1/6 (b) ¼ (c) 1/3 (d) 1/8
    Answer : (d)
15. A random variable X has following probability distribution

X	-2	-1	0	1	2	3
P(X)	0.1	K	0.2	2K	0.3	K

    Then the value of K is

(a) 1 (b) 0.1 (c) 10 (d) 0.01

Answer : (b)

True/False:

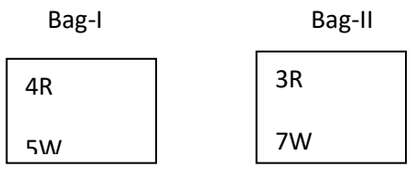
16. If P(A) =3/5, P(B) =3/10 and P(A∩B) = 1/5, then A and B are Independent (F)
17. If A and B are mutually exclusive events then they will be independent (F)
18. Two independent events are mutually exclusive (F)
19. If A and B are independent events then P(A∩B) = P(A) . P(B) (T)
20. Mean of probability distribution is also called expectation (T)
21. If P(A) = 0.6 then $P(\overline{A})$ = 0.6 (F)
22. P(A∪B) = P(A) + P(B) – P(A∩B) (T)
23. A coin is tossed six times, the probability of obtaining 4 heads is 15/64 (T)
24. If P(A) = 3/7, P(B) = 7/5, P(A∩B) = 3/5 then A and B are not independent (F)

3 Marks Questions

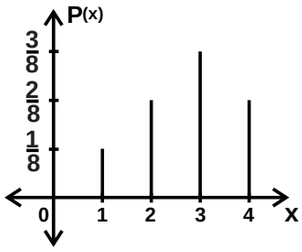
25. If A and B are independent events with P(A) = 0.3, P(B) = 0.4, find P(A∪B).
26. $P(\overline{A})$ = 0.6, P(B) = 0.2, P(B/A) = 0.5, find P(A∩B)
27. P(A) = 0.4, P(B) = 0.8 and P(B/A) = 0.6 find P(A∩B)
28. A coin is tossed two times. Find the probability distribution of number of tails.
29. Probability of solving specific problem independently by A and B are ½ and 1/3 respectively. If both try to solve the problem independently, find the probability that problem is solved.
30. If P(A) = 6/11, P(B) = 5/11 and P(A∪B) = 7/11 then find P(A∩B)
31. If A and B are two independent events and P(A) = ¼, P(B) = 1/3 find P(A∪B)

4 Marks Questions

32. Bag- I contains 4 red balls & 5 white balls and bag-II contains 3 red & 7 white balls as shown in figure. A bag is selected at random and a ball is drawn. Find the probability of getting a white ball.



33. The probability of solving a problem by A, B and C are P(A) = 1/3, P(B) = ¼, P(C) = 1/5. What is the probability that at least one of them will solve the problem?
34. If A and B are two independent events such that P(A) = ½, P(B) = p and P(A∪B) = 3/5, find p.
35. Find the mean of probability distribution given by following diagram:



36. Find the probability distribution of
    (i) Number of heads in two tosses of a coin
    (ii) Number of tails in four tosses of a coin
37. Find mean of number of heads in the three tosses of a coin.
38. A die is thrown 6 times. If getting an odd number is considered a success, what is the probability of 5 successes.

Linear Programming

Fill in the blanks from following options:

(Maxima and minima, optimal solution, linear programming problem, feasible region, constraints, unbounded, common, bounded)

1. The problems which seeks to maximize or minimize profit or loss is called_____.
   Ans : Liner programming problems.
2. The linear inequalities of a linear programming problem are called_____.
   Ans : Constraints .
3. The maximum or minimum value of linear function is called _____.
   Ans: optimal solution.
4. Feasible region is the _____region determined by all the constraints of a linear programming problem.
   Ans: common
5. The points within and on the boundary of a_____ represents feasible solution.
   Ans : feasible region
6. When the feasible region is bounded then Z _____.
   Ans : has maxima & minima

True/False

7. Subject to constraints $x+3y \leq 9$, $x \geq 0$, $y \geq 0$, maximum value of $z = x+2y$ is 9 (T)
8. Subject to constraints $2x+y \leq 4$, $x \geq 0$, $y \geq 0$, minimum value of $z = 2x+3y$ is 8 (F)
9. Subject to constraints $x+y \leq 4$, $x \geq 0$, $y \geq 0$, maximum value of $z = 3x+4y$ is 16 at the point (0,4) (T)
10. Minimum value of $z = 200x+500y$ subject to constraints $x+2y \geq 10$, $x \geq 0$, $y \geq 0$, is 2500. (F)
11. Subject to constraints $x+y \leq 50$, $x \geq 0$, $y \geq 0$, maximum value of $z = 4x+y$ is 200. (T)
12. When the feasible region is bounded, then Z has both maximum and minimum. (T)
13. Any point outside the feasible region is called an infeasible solution. (T)

Match the following:

14. (i) Max value of z = x+y subject to (p) 8

constraints x+y+2< 0, x ≥ 0, y ≥ 0 is
(ii) Max value of z = 2x+3y
subject to constraints x+2y ≤ 4,
x ≥ 0, y ≥ 0 is

(q) 3

(r) Does not exist

Ans [(i) - (r), (ii) - (p)]

15. (i) Minimum value of z = x+y
subject to constraints x+y+1< 0,
x ≥ 0, y ≥ 0 is
(ii) Minimum value of z = 3x+2y
subject to constraints 2x+y ≤ 8,
x ≥ 0, y ≥ 0 is

(p) 12

(q) -1

(r) Does not exist

Ans [(i), - (r), (ii) - (p)]

16. (i) Maximum value of z = 2x+y subje
constraints x+y≤ 3, x ≥ 0, y ≥ 0 is
(ii) Maximum value of z = x+3y
subject to constraints x+y≤ 8,
x ≥ 0, y ≥ 0 is

(p) 24

(q) 6

(r) 3

Ans :[(i) - (q), (ii) - (p)

17. (i) Minimum value of z = x+2y
subject to constraints x+y≤ 5,
x ≥ 0, y ≥ 0 is
(ii) Minimum value of z = 3x+y
subject to constraints x+y≤ 3,
x ≥ 0, y ≥ 0 is

(p) 3

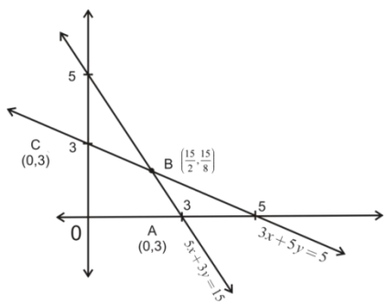
(q) 9

(r) 5

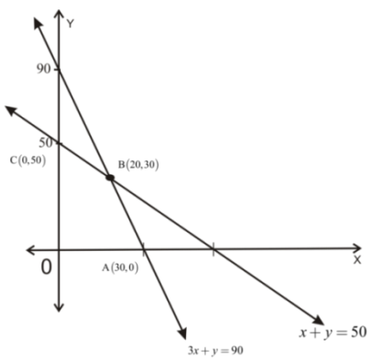
Ans :[(i) - (r) (ii) -(p)]

4 Marks Questions **:**

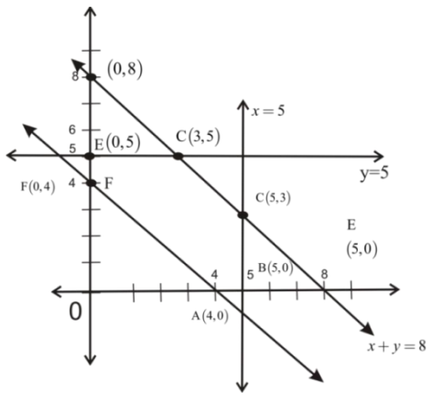
18. Shade the feasible region in the given figure subject to constraints $5x+3y \leq 15$, $3x+5y \leq 15$, $x \geq 0, y \geq 0$ also maximize $z = 8x+16y$ for the given graph.



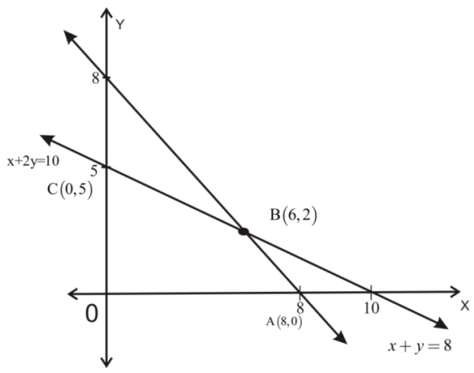
19. Maximize z=4x+y for the given graph subject to constraints $x+y \leq 50$, $3x+y \leq 90$, $x \geq 0$, $y \geq 0$ Also shade the feasible region is given graph.



20. Minimize $Z = x - 7y$ the given graph. shade feasible region. Subject to constraints $x \geq 0$, $y \geq 0$, $x+y \leq 8$, $x \leq 5$, $y \leq 5$, $x+y \geq 4$.



21. Minimize $z = 3x + 2y$ from your graph subject to constraints x+y£ 8, $x + 2y £ 10$, $x ³ 0 y ³ 0$



22. Shade the feasible region to the given figure subject to constraints $x + 2y ³ 10$, $3x + 4y £ 24$, $x ³ 0$, y ³ 0 . Also minimize $z = 200x + 500y$.